Proposal for a

Thesis in the Field of

Mathematics for Teaching

In Partial Fulfillment of the Requirements

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1. Tentative Title: Feasibility study for teaching geometry and other topics using three-dimensional printers

2. Abstract

Basic Euclidean geometry, with its straight lines, assumed parallels, and perfect circles works very well on paper and in the abstract. A student can learn how two points create a line, and a two lines lead to plane, all on paper. The third dimension requires some awkward drawings and a leap of faith. Students must understand that you’ve gone from squared to cubed, whether you look at a basic cube, a sphere, or a cylinder.

You can bring in dice and ping-pong balls or just look around the room to demonstrate objects in three dimensions. Some classes demonstrate three dimensional objects by folding papers and tediously taping together tabs. Most of these types of learning do not allow the students to grapple with the mathematics behind an object or to really work with dimensions or think about what happens when you change or combine shapes.

You can now design objects in computer aided drawing projects (CAD) and work with a developing line of hobbyist three-dimensional printers. The combination of software and printers allows students to design their own objects to explore dimension and design their own products or ideas. Until recently, the cost of both printer and software has been prohibitive, but hobby sites have developed their own inexpensive lines of printers and there are several free tools available to design in three dimensions. This thesis is a feasibility study to explore projects in geometry, trigonometry, and calculus on low-cost 3D printers. The project will include several test lessons, with a goal of testing some or all of the lessons with local high school students.

3. Thesis Project Description

The Feb 10, 2011 edition of *The Economist* ran a cover story *Print me a Stradivarius: How a new manufacturing technology will change the world*. In it, it describes a developing technology for designing objects on a computer and then having them slowly built, or “printed”, using a three-dimensional printer that drips material, slowly building what was designed on screen. The idea is that if everyone had materials for basic designs, everyone will be able to print items out at home, instead of at a factory that redistributes items through a store. Three-dimensional printing might completely change how the world does business. Amazingly, one company, EOS, was able to print a playable violin.

Some research speculates on the skills students need for future technology. Forman & Steen suggest that students need better numeracy, that is, how they understand numbers: “Many basic mathematical skills are essential for numeracy, including arithmetic, percentages, ratios, simple algebra, measurement, estimation, logic, data analysis, and geometric reasoning. But so too are other concepts not normally emphasized in traditional school mathematics:

- estimating tolerances and errors;
- simulating complex systems on computers;
- using flowcharts for planning and management;
- drawing inferences appropriately (statistical, scientific, logical);
- presenting data-based arguments by using modem computer tools;
- thinking, visualizing, and calculating in three dimensions.”

This project aims to educate students on understanding how to work in three dimensions, but the work they might do could expand into understanding tolerances and errors (the Wikipedia article on 3D printing states that, as with graphics, 3D printers have a X-Y resolution in DPI), computer simulations of what they will build, managing a project, and so on. This type of education can start at a very basic level with small children building their first cubes, to a sophomore geometry class as students learn to take the equation of a circle and build spheres and cylinders as they expand to three-dimensional thinking, to trigonometry as students discover curves.

Cliff, O’Malley, and Taylor all speculate that in 10-20 years, UK schools might all have 3D printers to create robots and electronics (19). Several engineering schools have already started to use 3D printers as part of their design courses, and the printers are starting to be used in k-12 education. This thesis aims to focus on using the 3D printers as tools that can be used to visualize three-dimensional geometry and trigonometry projects. It will include a component to analyze designs before and after printing in order to anticipate and correct mistakes. In addition, as many of the engineering schools using this technology are using printers that cost $15000 or more, this thesis will also explore what a school can do on a more limited budget.

For the project, I plan to obtain one of the 3D hobby printers. I am hoping to avoid a lengthy construction process and keep costs to under $3000. Possible printers are Botmill and MakerBot, both of which have kits available for under $1500. I plan to self fund the project. After the project is complete, I might try to work with community students or to bring the project or possibly class to a place like Artisan Asylum in Somerville, MA that uses CAD and CNC router currently.

You can already view examples of what you can create with three dimensional printing. Peter Schmitt at the Ars Electronica/MIT Media Lab has created a printable clock. (http://www.coolbuzz.org/entry/3d-printed-clock-by-peter-schmitt/). Ulrich Schwanitz designed a Penrose triangle illusion using a 3D printer, a task thought to be impossible, and set off speculation about the legal issues surrounding three-dimensional, printable designs (Masnick). And the Thingiverse site includes digital designs that anyone can print out themselves, or allows anyone to upload their own designs.

I am uniquely suited to work on this project. I have worked in several CAD programs, starting in a high school architecture course designing houses using MiniCAD, then studying AutoCAD and ProEngineer as part of coursework in mechanical engineering at the University of Massachusetts Amherst. As an engineering student, one of my favorite projects was designing a chess piece in AutoCAD and then using a lathe to actually cut it.

At Harvard Extension, I have taken courses that centered heavily around geometry, a subject I didn't appreciate in high school but that I have grown to love as I learn its history and its elegance.

I consider this work to have a potential entrepreneurial aspect to it, and I was one of the original founders of the entrepreneurship program at the University of Massachusetts Amherst, and continue to keep in contact with advisors to the program. I worked at the National Collegiate Inventors and Innovators Alliance, which funded student projects, often in engineering, to go toward commercialization. I currently work as a technical writer at Oracle, where I have team lead experience and can instruct on creating plans, timelines, and value propositions.

General outline

The thesis will begin with an overview of the 3D printer, including its history and evolution, comparisons to other technology for 3D modeling, and its previous and current use in classroom settings for all ages.

The thesis will then describe in detail lesson plans that incorporate the 3D printer. It will focus on geometry but also include lessons in trigonometry and calculus as time allows. The thesis will examine the current techniques of teaching dimensionality and topics on 3D objects and examine whether the 3D printing technology can improve understanding those topics.

Planning lessons

This thesis will build out possible lessons, mainly around high school geometry topics for 3D objects. In addition, it will outline possible lessons in other subjects, for example, looking at solids of revolution in calculus.

The course will be lessons centered around the following topics:

- General 3D printing history and basic engineering concepts.
- Geometry lessons.
- Trigonometry lessons.
- Calculus lessons.

The lessons listed are sample lessons and might change. The lessons described could work either for individuals or groups of students, particularly the project lessons.

For the mathematics-based lessons, there will be a discussion and overview for the lesson, a “how to” section for how to use the drawing software, and a list of questions to answer. I will design step-by-step instructions for the drawing and printing portions. After the first lesson, there will be chances for students to use the printer.

After an object is printed, students should be able to manipulate and measure the physical shape. They can weigh the object or a series of similar objects and make comparisons. For example, if you double the length of a cube’s sides, what happens to the weight of the cube? How does that relate to the volume?

Because many hobby-level printers can produce objects using soft materials, students should also be able to take their objects apart. For example, how would you break down an octohedron? How do you measure its volume? What is the total area of its sides?

The following sections outline some possible lessons to be developed that use 3D printing technology to illustrate concepts for solids.

General engineering lessons:

- How items are created, and how to break down standard items into various shapes.
- Basic engineering fundamentals, including design, materials, and so on.
- An introduction to the 3D printer, including a discussion of the benefits and advantages of 3D printing, as well as the drawbacks.
- How to draw in a 3d drawing tool, first learning about 2d drawing and extending to 3d drawing.
- How to work with a 3d printer, including examples of items that can be printed.
- Troubleshooting images and 3D models.

Geometry lessons:

- Create a lesson to learn about 2-dimensional objects, including circles, squares, and triangles.
  - Discuss the equations for each shape for area, perimeters, and so on.
  - Design these objects and talk about what happens as you change the shape of objects.
- Create several lessons to learn about 3-dimensional objects, including cubes, cylinders, spheres, cones, pyramids, and so on.
  - Have students or teams create the various shapes and discuss their process.
  - Print at least one student’s or one student group’s object for each item.
- Create a lesson around the platonic solids, including:
  - How to design the solid.
  - How to measure volumes.
  - Cut up the solids into pieces, as printers can produce “soft” models.
  - Discuss symmetry.
- Discuss geometry in terms of amount of material and cost of material.

- Discuss combining shapes, how area changes when you combine shapes, subtract shapes, and so on.
- Create a series of projects with increasing complexity, beginning with single shapes and ending with a more open-ended design project where students can learn:
- Combining several types of solids.
    - Removing a solid area in the middle of a shape and considering the volume and cost savings.
    - How to create a quality design.
    - How to troubleshoot a design that isn't working.

Trigonometry lessons:

- The same general lessons about engineering, drawing, and cost apply for trigonometry.
- Create a trigonometry lesson, for example, look at rotating arcs.

Calculus lessons:

- The same general lessons about engineering, drawing, and cost apply for calculus.
- Create a calculus lesson to discuss the volume of a sphere.
- Create a calculus lesson to discuss solids of revolution and their volumes.
- Create a calculus lesson to discuss minimizing and maximizing cost of a particular object.
- Create an open-ended project where students can combine standard geometry shapes and more complex curves. The objectives would be similar to the geometry design project.

Testing the lessons

After researching and developing the geometry lessons, I will attempt to identify and teach a sample group of students. I will conclude the thesis with findings from the test and recommendations for teachers who are considering creating and teaching mathematics lessons for 3D printers.

Since I am looking for detailed feedback, I think that this course would be suited to a smaller classroom of 8-14 students. Testing might even be possible with a smaller number of students. I expect students to work in pairs, as even in college this type of work seems to improve with more points of view and ideas.

The practice lessons will be situated after I outline the geometry section, so the students should be actively studying geometry. Ideally they will have already learned some aspects of three-dimensional mathematics, but at a minimum they should understand calculating the area of circles, squares, and triangles.

I expect that the students will need a few classes to learn the software and print sample objects, and it would be good if they are able to complete a simple, open-ended project on their own. I would expect several sessions and for students to work on their own outside of class before we could determine if this is successful. At the end, or possibly throughout, students will be given the opportunity to give feedback and comment on what they learned and what they found confusing in the lessons.

4. Work Plan

July/August

- Select and purchase printer and software (done).
- Document history of the 3D printer (one week).
- Include brief history of other engineering methods and their limitations (one week).
- Research use in schools or home schools (one week).
- Choose geometry problems, attempt to identify items suited to 3D printing (one week).

September

- Create a series of introductory geometry lessons for students to design basic geometry shapes in 3D drawing software (four weeks).
- Research introduction to the geometry of these shapes.
- Research going from 2D to 3D shapes and how students are usually taught.
- Design procedures for creating basic shapes, such as cubes, cylinders, pyramids. Consider designing procedures for all platonic solids. Document instructions and goals.
- Create a list of questions about designing the objects.

October

- Create a more advanced geometry lesson for students to design other shapes and to design their own shape using multiple shapes (two weeks).
- Locate students for lessons and prepare with teacher.
- Design a one-hour demo (two weeks).

November

- Consider on the main issues of the mathematics behind dimensions and how this technology helps clarify those lessons (two weeks).
- Document the mathematics behind the projects, including different ways of looking at volume in different shapes (two weeks).

December

- Demonstrate the project, possibly in a home school or honors class. Add comments and updates to the thesis based on questions in the demo (4-8 weeks).

- Create a trigonometry lesson around rotating a curve, for example, a chess piece (two weeks).

January

- Create a series of calculus lessons around spheres and rotated curves.
- Research introduction to the calculus of these shapes.
- Design procedures for creating the basic shapes, and incorporate the mathematics into the procedure.
- Discuss the calculus around cost and the volume of the shape versus other factors.
- Design an open-ended lesson for students to create their own project out of composite shapes.

February

- Document the mathematics behind the projects, including different ways of looking at volume in different shapes (two weeks).
- Document the engineering considerations and mathematics, including the strength of materials, cost, and so on (two weeks).
- Complete thesis writing.

5. Glossary

3D printing: “a form of additive manufacturing technology where a three dimensional object is created by laying down successive layers of material” (http://en.wikipedia.org/wiki/3D_printer).

Numeracy: the ability to reason with numbers and other mathematical concepts. (http://en.wikipedia.org/wiki/Numeracy)

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